Exercises from Jury and Horton, Soil Physics, 6th edition

Questions (answers given below)

3.2 A saturated soil column (Fig. 3.22) contains two soil layers, each 10 cm thick, with sand of $K_s = 10 \text{ cm } h^{-1}$ underneath loam of $K_s = 5 \text{ cm } h^{-1}$. The bottom of the soil column is open to the atmosphere ($p = 0$). At $t = 0$, water of height $d = 10$ cm above the top of the column is ponded on the surface.

(a) Assuming steady state, calculate the flux through the column.

(b) Assuming steady state, calculate the water pressure at the sand–loam interface (point A).

(c) If this were a falling-head permeameter, what would probably happen as d decreased from 10 cm to zero?

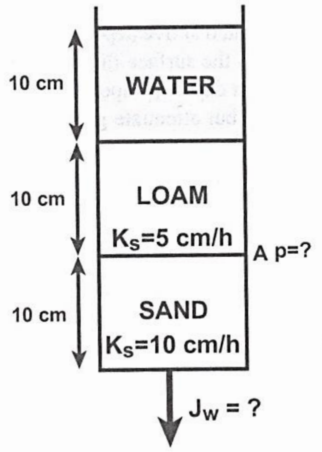

Figure 3.22 Layered soil column in Problem 3.2.

3.3 A soil column contains 50 cm of sand over 50 cm of clay (Fig. 3.23). A piezometer at $z = 50$ cm measures the hydrostatic pressure head $p = 50$ cm at the interface. The column is saturated and 20 cm of water is ponded on the top while the bottom is open to the atmosphere ($p = 0$). The steady measured flux rate is $J_w = -20 \text{ cm h}^{-1}$. Calculate the saturated hydraulic conductivity of the sand and the clay and the effective conductivity of the column.

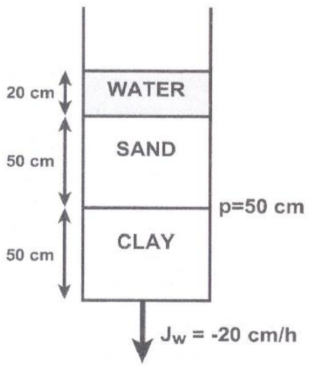

Figure 3.23 Layered soil column in Problem 3.3.

3.4 A cylindrical soil column of 100-cm² cross-sectional area and 50-cm height is filled with homogeneous soil that is saturated, and 10 cm of water is continuously ponded on it. The steady-state volume flow rate Q through the soil is $1000 \text{ cm}^3 \text{ h}^{-1}$ (downward).

(a) Calculate the steady-state flux through the column.

(b) Calculate the saturated hydraulic conductivity of the soil.

3.6 A clay lens of saturated hydraulic conductivity $K_c = 0.1$ cm day^{-1} is sandwiched somewhere inside a soil column of height 100 cm that is otherwise filled with sand of $K_s = 200$ cm day^{-1}. A height of water $d = 10$ cm is ponded above the surface.

(a) If the clay lens were not present, what would the steady flux rate be?

(b) If the actual flux rate is $J_w = -15$ cm day^{-1}, how thick is the lens?

(c) Calculate the pressure at the clay–sand interface for two special cases:

(i) Clay on top of the sand

(ii) Clay on the bottom of the sand

(d) Discuss the physical implications of your results.

3.11 A falling-head permeameter is constructed by ponding 20 cm of water at $t = 0$ above a saturated soil column of height $L = 100$ cm. The ponded water, which is held above the column in a chamber of the same area as the column, falls to 0 cm in 1 h.

(a) Calculate K_s for this column.

(b) Next repeat the analysis approximately by pretending that the water flux through the column was constant and equal to $J_w = -20$ cm h^{-1} and that the height of ponded water was held at 10 cm for the entire experiment.

(c) Compare the result calculated by Darcy's law with these approximations to the exact solution and calculate the percentage of error.

3.12 Repeat Problem 3.11 for the case where the soil column is only 5 cm high. Explain the reason for the difference in these two results.

3.13 Calculate the effective hydraulic conductivity of a saturated soil column of length L that has a saturated hydraulic conductivity $K_s(z)$ that varies arbitrarily over its length. Show that (3.26) is a special case of the formula you derive.

Eq. (3.26), effective hydraulic conductivity for flow through a layered soil (*N* layers):

$$K_{eff} = \frac{\sum_{J=1}^{N} L_J}{\sum_{J=1}^{N} \frac{L_J}{K_J}}$$

Answers are given below

3.2 This problem is solved by creating an effective homogeneous column with the same hydraulic resistance as the layered column.

(a) The first step is to calculate the effective hydraulic conductivity using (3.26):

$$\frac{20}{K_{\text{eff}}} = \frac{10}{5} + \frac{10}{10} = 3 \longrightarrow K_{\text{eff}} = \frac{20}{3} \text{ cm day}^{-1}$$

Then Darcy's law is written across the effective column, which has a total hydraulic head of 30 cm at the top and 0 at the bottom. Thus,

$$J_w = -K_{\text{eff}} \frac{H_2 - H_1}{z_2 - z_1} = -\frac{20}{3}\frac{30}{20} = -10 \text{ cm day}^{-1}$$

(b) The water pressure is calculated by writing Darcy's law across the bottom half of the column:

$$J_w = -10 = -K_2 \frac{z_A + p}{z_A} = -10 \frac{10 + p}{10} \longrightarrow p = 0$$

(c) Since the pressure is exactly zero at the interface when the height of water above the column is 10 cm, as it decreases, p will become negative, eventually desaturating the column if the air entry suction is exceeded.

3.3 Since the pressure is given at the interface, the hydraulic conductivity of each layer can be calculated directly by Darcy's law. If we set $z = 0$ at the bottom (point 1), then $H_1 = 0$. At the interface (point 2), $H_2 = 50 + 50 = 100$ cm. At the top of the column (point 3), $H_3 = 100 + 20 = 120$ cm. Thus, applying Darcy's law across the sand layer, we obtain

$$J_w = -20 = -K_{\text{sand}} \frac{H_3 - H_2}{z_3 - z_2} = -K_{\text{sand}} \frac{20}{50} \longrightarrow K_{\text{sand}} = 50 \text{ cm h}^{-1}$$

Similarly, for the clay layer we obtain

$$J_w = -20 = -K_{\text{clay}} \frac{H_2 - H_1}{z_2 - z_1} = -K_{\text{clay}} \frac{100}{50} \longrightarrow K_{\text{clay}} = 10 \text{ cm h}^{-1}$$

Finally, the effective conductivity is obtained across the entire column as

$$J_w = -20 = -K_{\text{eff}} \frac{H_3 - H_1}{z_3 - z_1} = -K_{\text{eff}} \frac{120}{100} \longrightarrow K_{\text{eff}} = \frac{100}{6} \text{ cm h}^{-1}$$

Note that we can also obtain K_{eff} from

$$\frac{100}{K_{\text{eff}}} = \frac{50}{K_{\text{sand}}} + \frac{50}{K_{\text{clay}}}$$

3.4 The cross-sectional area $A = 100 \text{ cm}^2$, volume flow rate $Q = 1000 \text{ cm}^3 \text{ h}^{-1}$, column height $L = 50$ cm, and ponding height $d = 10$ cm.

(a) By definition, the flux $J_w = Q/A = -10 \text{ cm h}^{-1}$.

(b) By Darcy's law, $K_s = -J_w L/(L + d) = 8.33$ cm.

3.6 Let the thickness of the lens be x and the total column height L.

(a) Without the lens, we use Darcy's law:

$$J_w = -K_S \frac{d+L}{L} = -220 \text{ cm day}^{-1}$$

(b) If the actual flux is only -15 cm day^{-1}, we first calculate K_{eff} for the layered system by Darcy's law:

$$J_w = -15 = -K_{\text{eff}} \frac{d+L}{L} \longrightarrow K_{\text{eff}} = \frac{15 \times 100}{110} = \frac{150}{11} \text{ cm day}^{-1}$$

Now we imagine a three-layer column, consisting of sand with clay in the middle somewhere. Letting the top layer thickness be z, the clay layer x, and the bottom $100 - x - z$, we use the layered soil formula (3.26)

$$\frac{100}{K_{\text{eff}}} = \frac{z}{K_S} + \frac{100 - x - z}{K_S} + \frac{x}{K_C} = \frac{100 - x}{K_S} + \frac{x}{K_C} \longrightarrow x = 0.6837 \text{ cm}$$

(c) There are two possibilities:

(i) When the clay is under the sand, we write Darcy's law across the clay layer:

$$J_w = -15 = -K_C \frac{p+x}{x} \longrightarrow p = 101.86 \text{ cm}$$

(ii) When the clay is on top of the sand, we write Darcy's law across the clay layer:

$$J_w = -15 = -K_C \frac{110 - (100 - x + p)}{x} \longrightarrow p = -91.87 \text{ cm}$$

(d) In the second case the system will definitely be unsaturated underneath the clay layer.

3.11 **(a)** Let us first set up the problem symbolically, using d as the initial ponding height, L as the column height, and t as the elapsed time. Then by the permeameter formula (3.21), the exact answer is

$$K_s = \frac{L}{t} \log \frac{d+L}{L} = 18.23$$

(b) The approximate form of Darcy's law is

$$J_w \approx -\frac{d}{t} = -K_s \frac{d/2+L}{L} \longrightarrow K_s = \frac{dL}{(d/2+L)t} = 18.18$$

(c) These are remarkably close, for the simple reason that the hydraulic head gradient doesn't change very much (from 120 to 100) during the experiment, so that using the constant value of 110 is a good approximation.

3.12 In this case, the two formulas give 8.05 and 6.67 using the exact and approximate formulas. There is significantly more error this time because the hydraulic head gradient changes from 25 to 5 during the experiment, so that the constant value of 15 is not a good approximation.

3.13 For this problem, the flux equation may be written as

$$J_w = -K(z)\frac{dH}{dz}$$

We can rearrange this and integrate from $z = 0$, where $H = H_1$, to $z = L$, where $H = H_2 = H_1 + \Delta H$:

$$\int_{H_1}^{H_2} dH = \Delta H = -J_w \int_0^L \frac{dz}{K(z)}$$

We may rewrite this as

$$J_w = -K_{\text{eff}} \frac{\Delta H}{L}$$

where

$$\frac{L}{K_{\text{eff}}} = \int_0^L \frac{dz}{K(z)} dz$$

which is a generalization of (3.26). Now we consider the special case where

$$L = \sum_{J=1}^{N} L_J \qquad K(z) = K_J \quad \text{when} \quad \sum_{K=1}^{J-1} L_K < z < \sum_{K=1}^{J} L_K$$

so that the effective conductivity formula becomes

$$\frac{L}{K_{\text{eff}}} = \sum_{J=1}^{N} \frac{L_J}{K_J}$$